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On the QCD analysis of jet broadening

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Abstract: The perturbative all-order analysis of the jet-broadening B-distribution in the small-B region is carried out with single-logarithmic accuracy, which requires the control of both the sum of the moduli and the modulus of the sum of the transverse momenta of soft gluons. We confirm the master equation for the B-distribution derived by Catani, Turnock and Webber (CTW). Proper treatment of quark recoil is necessary at this accuracy. This effect was neglected in the CTW solution. We show that the answer can be expressed in terms of the CTW result but evaluated at a properly rescaled B value.

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Contents

1 Introduction

Interesting characteristic features of a given shape variable can be appreciated only in calculations to next-to-leading order. One of the most interesting variables is the jet-broadening B-distribution, introduced in Ref. [1], in which $2B$ is the sum of the moduli of the transverse momenta of all emitted particles with respect to the thrust axis in units of Q, the total final state mass.

Consider, in perturbative QCD, the radiation emitted in e^+e^- annihilation consisting of the primary quark and antiquark with 4-momenta p and $\bar{p}$ and the secondary partons k_i. Hereafter we shall consider small values of B so that the secondary partons are soft and the primary p and $\bar{p}$ belong to opposite hemispheres. To be specific we define the right hemisphere as the one containing the quark p.

The simplest variable is the single-jet broadening (the right-jet broadening B_R in what follows) defined by

$$2B_R \;=\; \sum_{i\in R} |\vec{k}_{ti}| \;+\; p_t\,, \qquad \vec{p}_t \;=\; -\sum_{i\in R} \vec{k}_{ti}\,. \tag{1.1}$$

Since the transverse momenta are taken with respect to the thrust axis, the total vector sum of transverse momenta in each hemisphere is zero. One introduces the total jet-broadening B_T, as the sum of right- and left-jet broadening ($B_T = B_R + B_L$), and the wide-jet broadening $B_W = \mathrm{Max}\{B_R, B_L\}$.

We consider first the B_R-distribution. The jet-broadening distribution for the right hemisphere is given in terms of the multi-parton emission distribution $d\sigma_n$ by

$$\begin{aligned} \frac{d\sigma}{\sigma(d\ln B_R)} &= \frac{d}{d\ln B_R}\, I_R(B_R)\,, \\ I_R(B_R) &= \sum_n \int \frac{d\sigma_n}{\sigma}\; \Theta\left(2B_R - \sum_{i\in R}\left|\vec{k}_{ti}\right| - \left|\sum_{i\in R}\vec{k}_{ti}\right|\right)\,. \end{aligned} \tag{1.2}$$

For small B_R, which corresponds to all the final-state partons having small transverse momenta, one can approximate $d\sigma_n$ as a product of two factors. The first is a coefficient factor which depends only on $\alpha_s(Q)$. The second is an evolutionary exponent, which describes the production of small-k_t partons off the primary quark-antiquark pair. Hence

$$I_R(B) = C(\alpha_s(Q))\Sigma_R(B,\alpha_s)\,. \tag{1.3}$$

The essential momentum scales in the coupling in Σ_R range from B_RQ to Q. The perturbative treatment that we shall pursue requires $B_RQ \gg \Lambda_{QCD}$. The accuracy of the perturbative treatment is limited by non-perturbative power corrections of relative order $(\ln Q)\Lambda_{QCD}/(BQ)$ which are treated elsewhere [2, 3].

If only one gluon is present then the quark and gluon transverse momenta are equal and opposite — hence the natural factor of two in the definition of B_R (1.1). But starting from two gluon emission, the situation already becomes significantly more complex, because the modulus of the quark transverse momentum depends not only on the moduli of the gluon transverse momenta but also on their relative angles. So in higher orders one has to control simultaneously the sum of the moduli of the momenta, and the modulus of their vector sum. It turns out that in the kinematical region where $\alpha_s \ln 1/B_R \ll 1$, one gluon has a transverse momentum much larger than that of the others, so that the quark contribution is still $\frac{1}{2}p_\perp \simeq \frac{1}{2}B_R$ and therefore easily accounted for [4]. On the other hand, in the region of extremely small B, $\alpha_s \ln 1/B_R \gtrsim 1$, the gluonic contribution to B_R comes from several gluons with comparable transverse momenta, so that the problem of an accurate treatment of the quark recoil becomes severe.

A perturbative analysis based on the all-order resummation of leading and next-to-leading logarithmically enhanced contributions to jet-broadening distributions was performed by Catani, Turnock and Webber (CTW) in [4]. Their result in the kinematical region $B \ll 1$ is based on the "exponentiation" of one gluon emission. The approach that they developed guarantees that in the corresponding exponent, all terms with $\alpha_s^m \ln^n 1/B, m \leq n$ are kept track of. At a given point, they made the approximation that the quark recoil contribution to B_R is always $\frac{1}{2}p_\perp = \frac{1}{2}B_R$. This simplifies the answer, but mistreats the $\alpha_s^n \ln^n 1/B$ terms, starting from $n = 2$. In this note we will show how to treat properly the quark recoil and thus how to improve the CTW prediction for the jet-broadening spectrum.

Before discussing our result we first recall the form of the double-logarithmic contribution. Here one can simplify the analysis by assuming $\frac{1}{2}p_t = \frac{1}{2}B_R$. Moreover one assumes that all the final-state partons are soft and collinear gluons, emitted independently. This small B_R contribution to the jet-broadening distribution is given by

$$\Sigma_R(B_R) = e^{-R_0(1/B_R)} \left(1 + \mathcal{O}\left(\alpha_s^n \ln^n 1/B_R\right)\right), \tag{1.4}$$

where $R_0(1/B)$ is the soft part of the gluon emission distribution (see Appendix A) integrated over the region $k_t > B_R$

$$R_0(1/B_R) \;=\; \int_{Q^2 B_R^2}^{Q^2} \frac{dk_t^2}{k_t^2} \int_{k_t}^{1} dz\, \frac{\alpha_s(k_t) C_F}{2\pi}\, \frac{2}{z} \;=\; \frac{\alpha_s(Q) C_F}{\pi} \ln^2 1/B_R + \ldots \tag{1.5}$$

The limitation $k_t > B_R$ comes from the fact that for small B_R, to leading order the real emission takes place only for $k_t < B_R$ and here is cancelled by part of the virtual contribution. Thus only the virtual contribution remains in the region $k_t > B_R$. Taking into account the running of the coupling, R_0 becomes a series with terms of the form $\alpha_s^n \ln^{n+1} B$, while the neglected terms are one power of $\ln B$ down.

In our analysis, we intend to compute all single-logarithmic corrections, i.e. corrections of order $\alpha_s^n \ln^n B$. We show that to achieve this accuracy it suffices to "exponentiate" the next-to-leading order single-gluon emission formula [4] and to treat properly the quark recoil. The final result can be written in the form:

$$\begin{aligned}\Sigma_R(B_R) &= \left(\frac{e^{-\gamma_E R'}}{\Gamma(1-R')}\right) e^{-R(\lambda/2B_R)} \left(1 + \mathcal{O}\left(\alpha_s^n \ln^{n-1} 1/B_R\right)\right), \quad n \geq 2\,, \\ R' &\equiv -\frac{\partial R(1/B_R)}{\partial \ln B_R}\end{aligned} \tag{1.6}$$

where the radiator $R(\lambda/2B_R)$ is given by the one-gluon emission distribution, as in (1.5), but obtained from the next-to-leading order splitting function. The argument of the radiator is the jet-broadening B_R rescaled by a function $2/\lambda$ which has a single-logarithmic expansion, i.e. with leading terms of the order $\alpha_s^n \ln^n 1/B_R$. For B_R finite, λ tends to 2, while for small B_R it tends to 1. The function λ takes into account the effect of quark recoil. If one neglects quark recoil and puts for instance the quark transverse momentum at $p_t = B_R$, as done in [4], then one finds exactly the same expression (1.6) but with the function λ frozen at $\lambda = 2$.

Within our calculation we are also able to show the absence of the first non-logarithmic correction to $I(B)$ that comes from the running coupling at low scales, $\alpha_s(BQ)$. Though formally classified as being subleading, $\mathcal{O}\left(\alpha_s^n \ln^{n-1} 1/B\right)$, numerically it could be dangerous, since $\alpha_s(BQ)$ increases at small B.

It may be surprising that the accuracy in (1.6) can be achieved by using an independent multi-gluon emission distribution. The important point to realise is that subsequent gluon decay can be neglected to single-logarithmic order. This will be discussed in detail in the paper. However the following simple argument explains why. The B-spectrum contains a characteristic exponent of $\alpha_s \ln^2 1/B_R$. A value of B can be changed by a non-collinear non-soft gluon decay, with the transverse momenta of both offspring partons being $\mathcal{O}(BQ)$. Given the relative probability of such a decay, $\mathcal{O}(\alpha_s)$, we get a correction $B \to B(1+\alpha_s)$, which translates into $\alpha_s \ln^2(B(1+\alpha_s)) = \alpha_s \ln^2 B + \alpha_s^2 \ln B$, the latter being a negligible effect with a power of the log-enhancement factor smaller than that of α_s.

The paper is organised as follows. In section 2 we derive the B_R distribution using the independent emission distribution and by including the effect of quark recoil. We generalise the analysis to the total and wide jet-broadening distributions $\Sigma_T(B)$ and $\Sigma_W(B)$.

In section 3 we show that our result can be derived from the CTW equation obtained from the coherent branching process. To achieve the desired single-logarithmic accuracy one has to improve the treatment of recoil. We also verify that one can neglect the gluon branching process, thus justifying the use of the independent gluon emission approximation.

In section 4 we perform the actual evaluation of $\Sigma(B_R)$ to the required accuracy.

In section 5 we present the results of numerical analysis. We compare the final result with $\mathcal{O}(\alpha_s^2)$ numerical results obtained with the EVENT2 program [5]. We give all the necessary information to be able to carry out the matching with the fixed order perturbative results. We show the difference between our result and that of CTW, and compare the matched and non-matched calculations.

In section 6 we discuss, comment and look forward.

2 Independent gluon emission

To compute jet broadening to single-logarithmic accuracy, it is enough to use the independent gluon emission distribution from the primary quark-antiquark pair. This is given in terms of the $q \to q + g$ splitting function to next-to-leading accuracy.

The independent gluon emission distribution is

$$dw_n = \frac{1}{n!} \prod_{i=1}^{n} \frac{d^2 k_{ti}}{\pi k_{ti}^2} dz_i \; 2P_{qq}[\alpha_s, z_i] \; \Theta(z_i - k_{ti}) \, V \,, \qquad (2.1)$$

where the factor 2 takes into account the fact that, for each gluon, the integration domain includes both the right and left hemispheres. The quark splitting function is given, in the

$\overline{\text{MS}}$ scheme by

$$\begin{aligned} P_{qq}[\alpha_{\overline{\rm MS}}, z] &= C_F \frac{\alpha_{\overline{\rm MS}}(k_t)}{2\pi} \frac{1+z^2}{1-z} \left[1 \;+\; \frac{\alpha_{\overline{\rm MS}}}{2\pi} K\right] + \cdots \\ K &= C_A \left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9} n_f \,, \end{aligned} \tag{2.2}$$

for n_f flavours. The non-soft part of the quark splitting function, $1-z \sim 1$, produces a single-logarithmic contribution to the exponent, and must be kept, while the neglected part of the two-loop anomalous dimension generates negligible terms of order $\alpha_{\rm s}^2 \ln B$. The phase space region is $(1-z) > k_t/Q$ (see Appendix A).

We work in the physical (CMW) scheme [6] in which $\alpha_{\rm s}$ is the measure of the intensity of soft emission. We then have

$$P_{qq}[\alpha_{\rm s}, z] \;=\; C_F \frac{\alpha_{\rm s}(k_t)}{2\pi} \frac{1+z^2}{1-z}\,, \quad \alpha_{\rm s}(k_t) = \alpha_{\overline{\rm MS}}(k_t) \left(1 \;+\; \frac{\alpha_{\overline{\rm MS}}}{2\pi} K \;+\; \mathcal{O}\left(\alpha_{\overline{\rm MS}}^2\right)\right) . \tag{2.3}$$

To this order, the virtual correction V is given by

$$\ln V \;=\; -\int^{Q^2} \frac{dk_t^2}{k_t^2} \int_0^{1-k_t/Q} dz\, 2P_{qq}[\alpha_{\rm s}, z]\,. \tag{2.4}$$

For collinear-safe inclusive quantities one can integrate over k_t in (2.4) and k_{ti} in (2.1) down to zero.

The distribution (2.1) is normalised to unity: $\sum_n \int dw_n = 1$, where the transverse momentum of each real gluon is integrated up to Q. The independent emission distribution is valid only for small k_{ti}. It simplifies the treatment but mistreats the non-logarithmic region of large transverse momenta, $k_t \sim Q$, both in real and virtual terms. This is compensated by the factor $C(\alpha_{\rm s})$ in (1.3). For small B values this factor is B-independent. The B-dependence is embodied into the Σ factor,

$$\Sigma_R(B) = \sum_n \int dw_n \;\Theta\left(2B - \sum_{i\in R}\left|\vec{k}_{ti}\right| - \left|\sum_{i\in R} \vec{k}_{ti}\right|\right) . \tag{2.5}$$

To make use of the factorisation structure of the multi-gluon matrix element (2.1) we introduce the Mellin representation of the Θ-function

$$\Theta\left(2B - \sum_{i\in R}\left|\vec{k}_{ti}\right| - \left|\sum_{i\in R} \vec{k}_{ti}\right|\right) \;=\; \int \frac{d\nu}{2\pi i\nu}\, e^{2\nu B}\, \frac{d^2p_t\, d^2b}{(2\pi)^2}\, e^{-i\vec{p}_t\cdot\vec{b}}\, e^{-\nu p_t} \prod_{i\in R} e^{-\nu k_{ti}} e^{-i\vec{b}\cdot\vec{k}_{ti}}\,. \tag{2.6}$$

Here ν is the Mellin variable conjugate to B, which runs parallel to the imaginary axis, to the right of $\nu = 0$. We have introduced the integration over the quark transverse momentum $\vec{p}_t$ and the constraint $\vec{p}_t = -\sum_{i\in R} \vec{k}_{ti}$ is implemented by the integration over $\vec{b}$. We obtain

$$\Sigma_R(B) \;=\; \int \frac{d\nu}{2\pi i\nu} e^{2\nu B}\, \sigma(\nu)\,, \tag{2.7}$$

where the contour lies to the right of all singularities of $\sigma(\nu)$ which is given by

$$\sigma(\nu) \;=\; \int \frac{d^2p_t d^2b}{(2\pi)^2} e^{-i\vec{p_t}\cdot\vec{b}} e^{-\nu p_t} \cdot e^{-\mathcal{R}(\nu,b)} \;=\; \int_0^\infty \frac{\nu b db}{(\nu^2+b^2)^{3/2}}\, e^{-\mathcal{R}(\nu,b)}\,, \tag{2.8}$$

with the radiator

$$\mathcal{R}(\nu,b) \;=\; \int \frac{d^2k_t}{\pi k_t^2} dz\; P[\alpha_s, z] \left[1 \;-\; e^{-\nu k_t} e^{-i\vec{b}\cdot\vec{k_t}}\right]. \tag{2.9}$$

In the CMW scheme the two-loop α_s^2 contribution to the quark splitting function (2.3) is regular at $z = 1$ and can be neglected because it produces corrections to (2.5) of order $\alpha_s^n \ln^{n-1} 1/B$. In the $\overline{\rm MS}$ regularisation scheme instead one should explicitly keep the infrared-singular contribution of order $\alpha^2_{\overline{\rm MS}}$ to the splitting function (2.2), in order to reach the desired accuracy, as done in [4].

Notice that one is completely inclusive with respect to gluons emitted in the left hemisphere. Thus their contribution cancels $\sqrt{V}$, i.e. half of the virtual corrections. The remaining $\sqrt{V}$ corresponds to the term 1 in the square bracket of (2.9).

Performing the azimuthal integration we obtain

$$\mathcal{R}(\nu,b) = \int_0^1 \frac{dk_t}{k_t}\, \phi(1/k_t) \left[1 - e^{-\nu k_t} J_0(bk_t)\right]\,, \tag{2.10}$$

with $\phi(1/k_t)$ the one-gluon radiation formula to next-to-leading accuracy

$$\phi(1/k_t) \;=\; 2\frac{\alpha_s(k_t) C_F}{\pi} \left(\ln\frac{1}{k_t} - \frac{3}{4}\right) \;+\; \mathcal{O}\left(\alpha_s^2(k_t)\right)\,. \tag{2.11}$$

In the leading double-logarithmic approximation $\Sigma_R(B)$ is obtained as follows. Since quark recoil is irrelevant to this order, we are free to set $p_t = B_R$, which corresponds to neglecting the b-dependence in the radiator, i.e. replacing $\mathcal{R}(\nu,b)$ with $\mathcal{R}(\nu,0)$ in (2.8). Then, for large ν, one approximates $[1-e^{-\nu k_t}] \to \Theta(k_t - 1/\nu)$ and further approximates the ν-integration by setting $\nu \to 1/B$. This leads to the leading order result (1.4).

Following CTW, from $\sigma(\nu)$ one deduces the total jet broadening $\Sigma_T(B)$ and the wide-jet broadening $\Sigma_W(B)$ distributions given, for small B, by

$$\Sigma_T(B) \;=\; \int \frac{d\nu}{2\pi i\nu} e^{2B\nu}\, \sigma^2(\nu)\,, \quad \Sigma_W(B) \;=\; \left(\int \frac{d\nu}{2\pi i\nu} e^{2B\nu}\, \sigma(\nu)\right)^2. \tag{2.12}$$

Characteristic values of the Laplace parameter ν and of the impact parameter b that determine the final answer for the B-distribution satisfy $\nu \cdot BQ \sim 1\,,\quad b \sim \nu$, hence, in the $B \ll 1$ kinematical region we have a large parameter $\nu Q \sim 1/B \gg 1$.

A systematic evaluation of both $\sigma(\nu)$ and $\Sigma(B)$ to the required accuracy will be considered later. First we discuss the connection to the approach of CTW, obtained in the framework of coherent branching.

3 Coherent branching

In [4] a technique for analysing $\Sigma(B)$ was developed, based on the evolution equation for the distribution $T_q(Q,\vec{k},P_t)$ of the variable

$$P_t = \sum_{i\in R}\left|\vec{k}_{ti}\right| + \left|\sum_{i\in R}\vec{k}_{ti}\right|$$

in a quark jet produced with vector transverse momentum $\vec{k}$. The final physical distribution is obtained by setting $\vec{k}=0$. The distribution $\sigma(\nu)$ in (2.8) is given by its Laplace transform at $\vec{k}=0$

$$\sigma(\nu) \;=\; \tilde{T}_q(Q,\vec{0},\nu)\,, \qquad \tilde{T}_q(Q,\vec{k},\nu) \;=\; \int_k^\infty dP_t\, e^{\nu(k-P_t)}\, T_q(Q,\vec{k},P_t)\,. \tag{3.1}$$

From the coherent branching picture one has the following evolution equation (see [4])

$$\begin{aligned}\tilde{T}_q(Q,\vec{k};\nu) = 1 + \int_0^Q \frac{d^2\tilde{q}}{\pi\,\tilde{q}^2}\int_0^1 dz\,P_{qq}[\alpha_s,z]\Big\{e^{\nu(|\vec{k}|-|z\vec{k}+\vec{q}_t|-|(1-z)\vec{k}-\vec{q}_t|)}\,\cdot\\ \tilde{T}_q(z\tilde{q},z\vec{k}+\vec{q}_t;\nu)\tilde{T}_g((1-z)\tilde{q},(1-z)\vec{k}-\vec{q}_t;\nu) - \tilde{T}_q(\tilde{q},\vec{k};\nu)\Big\}\,,\end{aligned} \tag{3.2}$$

where $\vec{q}_t \equiv z(1-z)\vec{\tilde{q}}$, $q_t = |\vec{q}_t|$ and $k=|\vec{k}|$. Here, $\tilde{T}_g$ is the corresponding distribution for a gluon jet. The first term in the curly brackets describes real parton splitting, while the second subtraction term accounts for the virtual effects.

Introducing the Fourier transform

$$\Gamma_i(Q,\vec{b},\nu) = \int \frac{d^2k}{2\pi}\,e^{i\vec{b}\vec{k}\,-\,\nu k}\,T_i(Q,\vec{k},\nu)\,, \quad i=q,g\,, \tag{3.3}$$

we find

$$\begin{aligned}\Gamma_q(Q,\vec{b},\nu) = N(\nu,b) + \int_0^{Q^2}\frac{d\tilde{q}^2}{\tilde{q}^2}\int_0^1 dz\,P_{qq}[\alpha_s,z]\\ \left\{\int\frac{d^2r}{2\pi}J_0(rz(1-z)\tilde{q})\,\Gamma_q(z\tilde{q},\vec{b}+(1-z)\vec{r},\nu)\,\Gamma_g((1-z)\tilde{q},\vec{b}-z\vec{r},\nu) \;-\; \Gamma_q(\tilde{q},\vec{b},\nu)\right\},\end{aligned} \tag{3.4}$$

where $N(\nu,b)$ is the Fourier transform of the inhomogeneous term

$$N(\nu,b) = \int\frac{d^2k}{2\pi}\,e^{i\vec{b}\vec{k}-\nu k} = \frac{\nu}{(\nu^2+b^2)^{3/2}}\,. \tag{3.5}$$

Since the dependence on the evolution parameter Q is contained in the upper limit of the $\tilde{q}$–integration, we consider the logarithmic derivative of (3.4):

$$\Gamma'_q(Q,\vec{b},\nu) \;\equiv\; \frac{\partial}{\partial\ln Q}\Gamma_q(Q,\vec{b},\nu)\,. \tag{3.6}$$

Then $\tilde{q}$ in the integrand gets replaced by Q and $q_t = z(1-z)Q$. It is straightforward to verify that the integral term possesses the damping factors $e^{-\nu\, z(1-z)Q}\, J_0(b\, z(1-z)Q)$, and therefore is concentrated at small values of $(1-z)$,

$$(1-z) \lesssim \min\left\{(\nu Q)^{-1}, (bQ)^{-1}\right\}. \tag{3.7}$$

As a result, any correction proportional to $(1-z)$, or $\ln z$, will produce a *power-suppressed* contribution $\sim 1/\nu Q \propto B \ll 1$. Taking into account that $r \sim b \sim \nu$, this allows us to approximate $q_t = z(1-z)Q \simeq (1-z)Q$, which correction is of the order of $\alpha_{\rm s}(Q)/\pi$ and will be absorbed into the coefficient function $C(\alpha_{\rm s})$ in (1.3). We can also replace in (3.4)

$$\Gamma_q(zQ, \vec{b} + (1-z)\vec{r}, \nu) \;\Rightarrow\; \Gamma_q(Q, \vec{b}, \nu)\,,$$

and

$$\Gamma_g((1-z)Q, \vec{b} - z\vec{r}, \nu) \;\Rightarrow\; \Gamma_g((1-z)Q, \vec{b} - \vec{r}, \nu)\,.$$

The accuracy of these approximations is of the order of the relative corrections $\alpha_{\rm s}/\pi \cdot B$. Notice that the first argument in Γ_g, generally speaking, cannot be expanded in $(1-z) \ll 1$ since the corresponding dependence is double-logarithmic.

By making these simplifications ($q_t = (1-z)Q$) we obtain

$$\frac{\Gamma_q'(Q, \vec{b}, \nu)}{\Gamma_q(Q, \vec{b}, \nu)} = 2\int_0^1 dz\, P_{qq}[\alpha_s, z]\left\{J_0(bq_t)\, e^{-\nu q_t}\, T_g(q_t, q_t, \nu) - 1\right\}. \tag{3.8}$$

The final solution reads

$$\begin{aligned} \tilde{T}_q(Q, 0, \nu) &= \int \frac{d^2b}{2\pi}\, N(\nu, b)\, e^{-S(\nu,b)}\,, \\ S(\nu, b) &= \int_0^{Q^2} \frac{d^2q_t}{\pi q_t^2} \int_0^{1-q_t/Q} dz\, P_{qq}[\alpha_s(q_t), z]\left[1 \;-\; e^{-\nu q_t + i\vec{b}\vec{q}_t}\, T_g(q_t, q_t, \nu)\right]. \end{aligned} \tag{3.9}$$

Neglecting gluon branching, $T_g \Longrightarrow 1$, we obtain the result of the previous section

$$\sigma(\nu) = \tilde{T}_q(Q, 0; \nu) = (2.8)\,. \tag{3.10}$$

To see that gluon branching, $T_g \neq 1$, is indeed negligible within our accuracy, we look at the correction to the exponent in (3.9):

$$\delta S(\nu, b) = \int_0^{Q^2} \frac{d^2q_t}{\pi q_t^2} \int_0^{1-q_t/Q} dz\, P_{qq}[\alpha_s(q_t), z]\, e^{-\nu q_t + i\vec{b}\vec{q}_t}\, \left[T_g(q_t, q_t, \nu) - 1\right]. \tag{3.11}$$

First we notice that the exponential factor forces $q_t \lesssim \nu^{-1}$. On the other hand, by examining the evolution equation (3.2) we observe that $T_g(q_t, q_t, \nu) - 1$ vanishes for $q_t \ll \nu^{-1}$. As a result, the q_t-integral is concentrated at $q_t\nu \sim 1$, $T_g(q_t, q_t, \nu) - 1 \sim \alpha_{\rm s}\, \delta(\ln(q_t\nu))$, and the correction amounts to

$$\delta S \;\sim\; \alpha_{\rm s}^2 \ln(\nu Q)\,,$$

with the single logarithmic factor emerging from the z-integration.

This first perturbative correction is due to quark $\to$ (quark + two gluons/$q\bar{q}$) pair splitting processes in which two secondary partons have similar emission angles and energies of the same order. It is easy to check that a correction of the same order originates from non-collinear two-parton production at *large* angles $\Theta \sim 1$, and, in particular, of the configuration of partons falling into opposite hemispheres. In the present treatment the left and right jets contribute independently to the event broadening and so such subleading contributions are not included.

4 Evaluation of $\Sigma(B)$

In this section we first evaluate the radiator $\mathcal{R}(\nu, b)$ and then perform the b-integral in (2.8) to evaluate $\sigma(\nu)$. For the sake of simplicity in this section we put $Q = 1$.

4.1 Radiator

To evaluate the radiator for large ν and b values we introduce the function

$$R(\mu) \;=\; \int_{\mu^{-1}}^{1} \frac{dk}{k}\phi(k^{-1})\,, \quad \mu = \tfrac{1}{2}(\nu + \sqrt{\nu^2+b^2})\,, \tag{4.1}$$

which corresponds to (1.5) with the next-to-leading splitting function. With account of the running coupling, it contains all terms of order $\alpha_s^n \ln^{n+1}\mu$ as well some of the essential $\alpha_s^n \ln^n \mu$ terms. We write

$$\mathcal{R}(\nu, b) = R(\mu) \;+\; \delta R(\nu, b)\,,$$

and study the correction

$$\delta R(\nu, b) \;=\; \int_0^{\mu^{-1}} \frac{dk}{k}\phi(k^{-1})\left(1 - e^{-k\nu}J_0(bk)\right) - \int_{\mu^{-1}}^{1} \frac{dk}{k}\phi(k^{-1})\, e^{-k\nu} J_0(bk)\,. \tag{4.2}$$

It is determined by the integration region $k \sim \mu^{-1}$ and can be written as ($\epsilon \to 0$)

$$\begin{aligned} \delta R(\nu, b) \;&=\; \int_0^1 \frac{dz}{z} z^\epsilon\, \phi(\mu/z) - \int_0^\infty \frac{dz}{z} z^\epsilon\, e^{-z\nu/\mu} J_0(bz/\mu)\, \phi(\mu/z) \\ &= \int_0^1 \frac{dz}{z} z^\epsilon\, \phi(\mu/z) - \int_0^\infty \frac{dz}{z} z^\epsilon\, e^{-z\nu/\mu}\, \phi(\mu/z) - \int_0^\infty \frac{dz}{z}\, e^{-z\nu/\mu}\left(J_0(bz/\mu) - 1\right)\phi(\mu/z)\,, \end{aligned} \tag{4.3}$$

where we have neglected contributions of order of $\exp(-\mu)$.

To evaluate δR we expand $\phi(\mu/z)$ in powers of $\ln z$

$$\phi(\mu/z) = R'(\mu) - R''(\mu)\, \ln z + \ldots$$

with

$$\begin{aligned} R'(\mu) &\equiv \frac{d}{d\ln\mu} R(\mu) \;=\; \phi(\mu)\,, \\ R''(\mu) &= \frac{d}{d\ln\mu}\phi(\mu) \;=\; 2C_F \frac{\alpha_s(1/\mu)}{\pi} + \mathcal{O}\left(\alpha_s^2 \ln\mu\right)\,, \\ R'''(\mu) &= \mathcal{O}\left(\alpha_s^2 \ln\mu\right)\,. \end{aligned} \tag{4.4}$$

We have that $R'''(\mu)$ is beyond our accuracy since it would lead to corrections to $\Sigma(B)$ of order $\alpha_{\mathrm{s}}^n \ln^{n-1} 1/B$. In $R''(\mu)$ we keep the correction of the order of the coupling at the reduced scale $\alpha_{\mathrm{s}}(1/\mu)$ and neglect its higher powers in α_{s}. To our accuracy we could neglect the scale $1/\mu$ in α_{s}. However we shall keep track of the non-logarithmic corrections proportional to $\alpha_{\mathrm{s}}(1/\mu)$ that emerge in the course of approximate evaluation of the b- and ν-integrals. This enables us to guarantee that no first-order correction $\alpha_{\mathrm{s}}(BQ)$ is present in the final answer.

The contribution to $\delta R(\nu, b)$ proportional to $R'(\mu)$, the leading order correction, is given by

$$\delta R^{(1)} = R'(\mu) \cdot \gamma_E \,. \tag{4.5}$$

Having chosen a different definition of μ would have resulted in an additional logarithmic contribution.

The non-logarithmic correction, of order $R''(\mu) \sim \alpha_{\mathrm{s}}(1/\mu)$, is

$$\delta R^{(2)} = R''(\mu) \cdot \Delta(\nu, b) \,, \tag{4.6}$$

where (see Appendix B)

$$\begin{aligned} \Delta(\nu, b) &= \left\{ \tfrac{1}{2} \left(\ln \frac{\mu}{\nu} + \gamma_E \right)^2 + \tfrac{1}{2}\psi'(1) + \ln \mu \ln \frac{\nu}{\mu} + \; c(\nu, b) \right\} , \\ c(\nu, b) &= \int_0^\infty \frac{dx}{x} \ln x e^{-\nu x} \left(J_0(bx) - 1 \right) \;, \end{aligned} \tag{4.7}$$

We have then

$$\mathcal{R}(\nu, b) = R(\mu) + \delta R(\nu, b) = R(\mu e^{\gamma_E}) + R''(\mu) \cdot (\Delta(\nu, b) - \tfrac{1}{2}\gamma_E^2) \; + \mathcal{O}\left(R'''\right) . \tag{4.8}$$

Expanding in $\ln \mu/\nu$ we obtain

$$\mathcal{R}(\nu, b) = R(\bar{\nu}) + R'(\bar{\nu}) \; \ln \frac{\mu}{\nu} + R''(\bar{\nu}) \cdot \bar{\Delta} \left(\frac{\mu}{\nu} \right) \; + \mathcal{O}\left(R'''\right) . \tag{4.9}$$

where

$$\bar{\nu} \equiv \nu \, e^{\gamma_E} \,, \tag{4.10}$$

and

$$\bar{\Delta}(\mu/\nu) = \tfrac{1}{2}\psi'(1) + \gamma_E \; \ln \frac{\mu}{\nu} - \ln \nu \ln \frac{\mu}{\nu} + c(\nu, b) \,, \tag{4.11}$$

is a function of the ratio μ/ν and has no $\ln \nu$ terms (see Appendix B).

4.2 $\sigma(\nu)$

Now we substitute (4.9) into (2.8) and performing the b-integral to evaluate $\sigma(\nu)$. Changing the integration variable to $y = \sqrt{\nu^2 + b^2}$ we obtain ($\mu/\nu = \frac{1}{2}(1+y)$)

$$\sigma(\nu) = e^{-R(\bar{\nu})} \int_1^\infty \frac{dy}{y^2} \left(\frac{1+y}{2}\right)^{-R'(\bar{\nu})} \left(1 - R''(\bar{\nu})\, \bar{\Delta} \left(\frac{1+y}{2}\right) + \mathcal{O}\left(R'''\right)\right). \tag{4.12}$$

To calculate the main contribution we introduce the rescaling function $\lambda(R')$

$$(\lambda)^{-R'} = \int_1^\infty \frac{dy}{y^2} \left(\frac{1+y}{2}\right)^{-R'}. \tag{4.13}$$

To estimate the correction $R'' \sim \alpha_s$ to first order we can neglect the exponent R' since $\bar{\Delta}$ is of order one (it has no $\ln\nu$ contribution). It equals (see Appendix B)

$$\int_1^\infty \frac{dy}{y^2}\, \bar{\Delta} \left(\frac{1+y}{2}\right) = \tfrac{1}{2}(\psi'(1) + \ln^2 2)\,. \tag{4.14}$$

Finally we arrive at

$$\begin{aligned} \sigma(\nu) &= e^{-R(\bar{\nu})}\ (\lambda)^{-R'(\bar{\nu})}\ \left[1 - R''(\bar{\nu})\, \tfrac{1}{2}(\psi'(1) + \ln^2 2) + \mathcal{O}\left(R'''\right)\right] \\ &= e^{-R(\bar{\nu}\lambda)}\ \left[1 - \tfrac{1}{2}R''(\bar{\nu})\,\psi'(1) - \tfrac{1}{2}R''(\bar{\nu})(\ln^2 2 - \ln^2\lambda)\ + \mathcal{O}\left(R'''\right)\right], \end{aligned} \tag{4.15}$$

Since λ is a function of $R'(\bar{\nu})$, the leading terms of its expansion are $\alpha_s^n \ln^n \bar{\nu}$. For small values of $R'(\bar{\nu})$, the function $\lambda(R')$ is close to 2,

$$\lambda(R') = 2 - R'(\bar{\nu}) \left(\frac{\pi^2}{6} - 2\ln^2 2\right) + \mathcal{O}\left(R'^2\right), \tag{4.16}$$

while for large values of $R'(\bar{\nu})$ it approaches unity:

$$\lambda(R') = 1 + \frac{\ln(R'(\bar{\nu})/2)}{R'(\bar{\nu})} + \mathcal{O}\left(R'^{-2}\right). \tag{4.17}$$

Perturbatively, λ is close to 2, see (4.16). Therefore the quantity $R''(\ln^2\lambda - \ln^2 2)$ in (4.15) is of order R''' and thus negligible. We can write

$$\sigma(\nu) = e^{-R(\bar{\nu}\lambda)} \left[1 - \tfrac{1}{2}\psi'(1)\, R''(\bar{\nu})\ +\ \mathcal{O}\left(R'''\right)\right]. \tag{4.18}$$

Notice that this estimate is uniform in R'. Indeed, for large values of R' the $\ln^2\lambda$ term in (4.15) vanishes, see (4.17). At the same time, the $\ln^2 2$ term disappears as well, since for large R' the integral (4.12) for the correction is concentrated near $y = 1$ where $\bar{\Delta}(1) = \frac{1}{2}\psi'(1)$, instead of (4.14).

The next-to-next-leading correction $R''(\bar{\nu}) \sim \alpha_s(Q/\nu) \sim \alpha_s(BQ)$ that we kept in (4.18) will cancel, in the first order, against a similar correction coming from evaluation of the ν-integral.

4.3 The ν-integral

Here we finally compute the integral over ν in (2.7) to obtain $\Sigma_R(B)$. To this end we expand $\sigma(\nu)$ around some point ν_0 in powers of $\ln(\nu/\nu_0) \equiv \ln t$. Introducing $\eta_0 \equiv \bar{\nu}_0 \lambda(R')$ we have, to the required accuracy

$$\Sigma_R(B) = e^{-R(\eta_0)} \int \frac{dt}{2\pi i\, t}\, e^{2\nu_0 Bt}\, t^{-R'(\eta_0)} \left[1 - \tfrac{1}{2}R''(\bar{\nu})\left(\ln^2 t + \psi'(1)\right) \;+\; \mathcal{O}\left(R'''\right)\right]. \quad (4.19)$$

Notice that the corrections coming from differentiation of $\lambda(R')$ in the argument of $R(\bar{\nu}\lambda)$ do not contribute: $\lambda \simeq (\alpha_{\rm s} \ln \bar{\nu})^n$, therefore $R' \cdot \lambda' \simeq \alpha_{\rm s}^m \ln^{m-1} \bar{\nu}$, escaping our resolution.

Now we have to choose the value of ν_0 such as to keep the characteristic value of $\ln t$ not large. The leading term is given by the following basic integral

$$\int_C \frac{dt}{2\pi i\, t}\, e^{2B\nu_0\, t}\, t^{-\gamma} = \frac{(2B\nu_0)^\gamma}{\Gamma(1+\gamma)}, \quad \gamma = R'(\eta_0)\,, \quad (4.20)$$

and we obtain

$$\Sigma_R(B) = e^{-R(\eta_0)} \left[1 - \tfrac{1}{2}R''(\eta_0)\left(\frac{d^2}{d\gamma^2} + \psi'(1)\right) \;+\; \mathcal{O}\left(R'''\right)\right] \frac{(2B\nu_0)^\gamma}{\Gamma(1+\gamma)}\,, \quad (4.21)$$

The correction proportional to $R''(\eta_0)$ becomes

$$\left(\frac{d^2}{d\gamma^2} + \psi'(1)\right) \frac{(2B\nu_0)^\gamma}{\Gamma(1+\gamma)} = \left(\left[\,\ln(2B\nu_0) - \psi(1+\gamma)\,\right]^2 + \psi'(1) - \psi'(1+\gamma)\right) \frac{(2B\nu_0)^\gamma}{\Gamma(1+\gamma)}\,. \quad (4.22)$$

Choice of ν_0. To ensure the smallness of the correction one has to choose ν_0 in such a way that $\ln(2B\nu_0)$ remains finite. By choosing

$$\nu_0 = \frac{1}{2B} e^{\psi(1)}\,, \qquad \bar{\nu}_0 \equiv \nu_0 e^{\gamma_E} = \frac{1}{2B}\,, \quad (4.23)$$

the first correction of order R'' in (4.19), given by (4.22), becomes uniformly small:

$$-\tfrac{1}{2}R''\,\left[\,(\psi(1) - \psi(1+\gamma))^2 + \psi'(1) - \psi'(1+\gamma)\,\right]. \quad (4.24)$$

It vanishes in the first order, that is for $\gamma = 0$.

The parameter η_0 reads

$$\eta_0 \equiv \bar{\nu}_0\, \lambda \;=\; \frac{\lambda}{2}\, B^{-1}\,,$$

and we arrive at

$$\Sigma_R(B) = e^{-R(\lambda/2B)}\, \frac{e^{-\gamma_E R'}}{\Gamma(1+R')} \equiv \Sigma_R^{(0)}(2B/\lambda)\,. \quad (4.25)$$

This first form of the answer is the CTW distribution with the important modification that the argument of the double-logarithmic radiator is $2B/\lambda$ instead of simply B. Since R' enters only in the prefactor, we can take $R' \;=\; R'(1/B)$.

We can further simplify the answer by expanding $R(\lambda/2B)$ in powers of $\ln\lambda/2$. Neglecting the contribution $\mathcal{O}(R'')$, we arrive at the second form of the answer,

$$\Sigma_R \;=\; \left(\frac{2}{\lambda}\right)^{R'} \cdot \frac{e^{-\gamma_E R'}}{\Gamma(1+R')}\, e^{-R(1/B)} = \left(\frac{2}{\lambda}\right)^{R'} \cdot \Sigma_R^{(0)}(B)\,, \tag{4.26}$$

where $R' \equiv R'(1/B)$. This is the CTW answer modified by the single-logarithmic factor $(2/\lambda)^{R'}$. The two forms of our answer, (4.25) and (4.26) are both correct (and equivalent) to single-logarithmic accuracy.

Finally our results for the total and wide jet broadenings (given in a form analogous to (4.26)) are

$$\Sigma_T \;=\; \left(\frac{2}{\lambda}\right)^{2R'} \cdot \frac{e^{-2\gamma_E R'}}{\Gamma(1+2R')}\, e^{-2R(1/B)} = \left(\frac{2}{\lambda}\right)^{2R'} \cdot \Sigma_T^{(0)}(B)\,, \tag{4.27}$$

and

$$\Sigma_W \;=\; \left(\frac{2}{\lambda}\right)^{2R'} \cdot \frac{e^{-2\gamma_E R'}}{\Gamma^2(1+R')}\, e^{-2R(1/B)} = \left(\frac{2}{\lambda}\right)^{2R'} \cdot \Sigma_W^{(0)}(B)\,, \tag{4.28}$$

respectively. As before, $R' \equiv R'(1/B)$. The value of $C(\alpha_{\mathrm{s}})$ in (1.3) is

$$C(\alpha_{\mathrm{s}}) = 1 + \sum_{n=1} C_n \alpha_{\mathrm{s}}^n\,, \qquad C_1 = \frac{C_F}{2\pi}\left(\pi^2 - \frac{17}{2}\right). \tag{4.29}$$

For consistency with the order at which we have performed the resummation, it is sufficient to know C only to first order in α_{s}.

5 Numerical analysis

5.1 Comparison with two-loop result

We compare the single-logarithmic result for the total and wide jet distribution ($i = T, W$)

$$\frac{d\sigma_i}{\sigma(d\ln B)} = \frac{d\,I_i(B)}{d\ln B}\,,$$

with the exact two-loop distribution obtained with the EVENT2 program [5].

To obtain the α_{s}^2 contribution from our calculation one needs to expand eqs. (4.27,4.28) up to second order in α_{s} and multiply by (4.29). At order α_{s}^2 the result should accurately reproduce the coefficient of terms $\alpha_{\mathrm{s}}^2 \ln^m 1/B$ with $m = 2, 3, 4$. To verify this, in figure 1, we plot as a function of B the differences

$$\delta_i^{\mathrm{s-log}}(B) \equiv \frac{d\,I_i^{2\ell}(B)}{d\ln B} - \frac{d\,I_i^{s-log}(B)}{d\ln B}\,,$$

$$\delta_i^{\mathrm{d-log}}(B) \equiv \frac{d\,I_i^{2\ell}(B)}{d\ln B} - \frac{d\,I_i^{d-log}(B)}{d\ln B}\,,$$

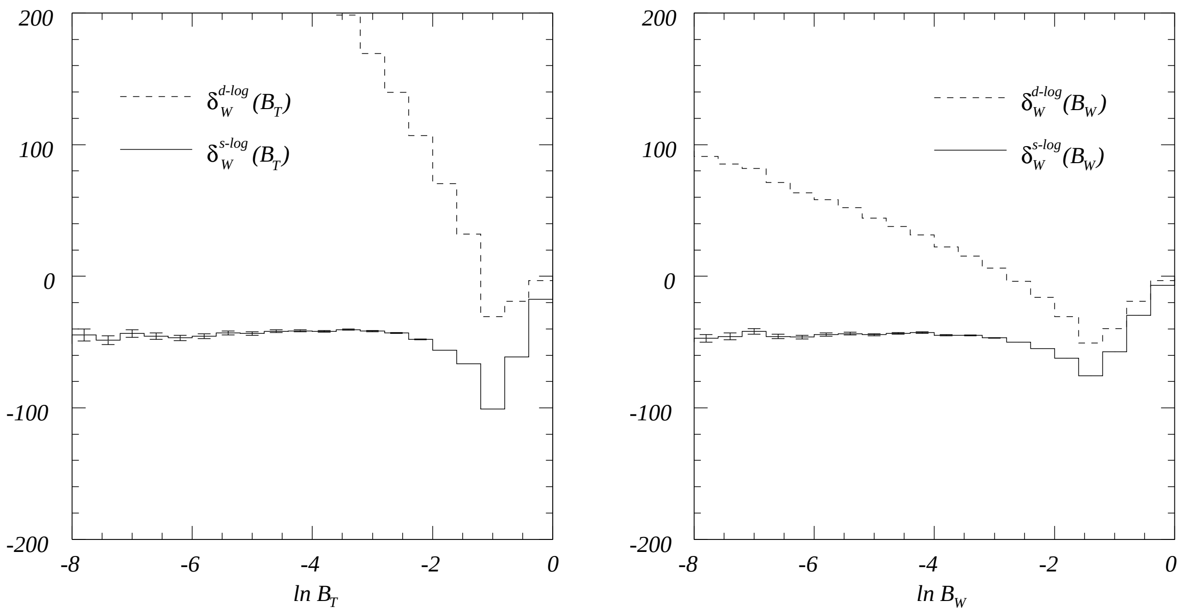

Figure 1: The coefficient of the $(\alpha_s C_F/2\pi)^2$ component of the difference, $\delta_i(B_i)$, between the numerical two-loop calculation of the B-distribution, performed with EVENT2, and the expansion to single and double logarithmic accuracies of the resummed result. Shown for the total and wide broadenings.

with $I_i^{2\ell}(B)$ obtained from EVENT2, $I_i^{\mathrm{d-log}}(B)$ obtained from the expansion of the resummed result to double-logarithmic accuracy ($\alpha_s^2 \ln^m 1/B$ with $m = 3, 4$) and $I_i^{\mathrm{s-log}}(B)$ to single logarithmic accuracy ($\alpha_s^2 \ln^m 1/B$ with $m = 2, 3, 4$). The comparison is made within the $\overline{\mathrm{MS}}$ scheme, and with I normalised to the Born cross section rather than the total cross section (as this is what is supplied by the EVENT2 program). We show only the coefficient of the $(\alpha_s C_F/2\pi)^2$ part of δ_i, as this is the only component modified by our new treatment of quark recoil.

The quantity $\delta_i^{d-log}(B)$ is of order $\alpha_s^2 \ln 1/B$, corresponding to the absence in $I_i^{\mathrm{d-log}}(B)$ of the $\alpha_s^2 \ln^2 1/B$ term. Meanwhile, $\delta_i^{s-log}(B)$ is of order α_s^2, indicating that in I we have correctly taken into account the $\alpha_s^2 \ln^2 1/B$ term.

5.2 Matching

When comparing with experimental data one often chooses to match the resummed calculation with a full second order calculation (as for example from the EVENT2 program). Here we give only a brief summary of two main matching procedures (log-R and R matching schemes) and refer the reader to [7] for detailed information.

For convenience we write

$$\ln \Sigma = \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \alpha_s^n L^m \tag{5.1}$$

$$= L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots , \tag{5.2}$$

with $L = \ln 1/B$. The resummation procedure provides g_1, g_2 and C_1. Given the resummed calculation, the full two-loop calculation contains information on G_{21} (the first term of g_3) and C_2 and on a "remainder" which does not necessarily exponentiate. Matching schemes put together these different parts with the principle ambiguity being the treatment of the remainder: the log-R scheme makes the approximation that it exponentiates, the R-scheme that it doesn't.

Here we provide the information that is needed for implementing the matching with our new calculation. We recommend the use of the second of the forms in (4.27,4.28), using as $\Sigma^{(0)}$ the equations (18–22) of [4]. The factor by which it is multiplied is

$$\left(\frac{2}{\lambda}\right)^{2R'} = \left[\int_1^\infty \frac{dx}{x^2}\left(\frac{1+x}{4}\right)^{-R'}\right]^2, \tag{5.3}$$

and one should use

$$R' = \frac{2\alpha_s C_F}{\pi}\frac{\ln(1/B)}{1-2\alpha_s\beta_0\ln(1/B)}, \tag{5.4}$$

with $\beta_0 = (11C_A - 2n_f)/12\pi$. This particular form ensures that the resummed calculation contains only g_1 and g_2 and no spurious g_3 type terms, as is required for correct matching.

In addition the matching schemes require G_{1m}, G_{2m}, C_1 and C_2 explicitly. G_{1m}, G_{23} and C_1 are correctly reproduced in [8].[1] The new analytic forms for G_{22} are

$$G_{22} = \frac{1}{(2\pi)^2}\left[-\left(32\,\ln^2 2 + \frac{8}{3}\pi^2\right)C_F^2 + \left(\frac{2}{3}\pi^2 - \frac{35}{9}\right)C_A\,C_F + \frac{2}{9}\,C_F\,n_f\right] \tag{5.5}$$

for the total broadening, and

$$G_{22} = \frac{1}{(2\pi)^2}\left[-(32\,\ln^2 2)\,C_F^2 + \left(\frac{2}{3}\pi^2 - \frac{35}{9}\right)C_A\,C_F + \frac{2}{9}\,C_F\,n_f\right] \tag{5.6}$$

for the wide broadening.

Finally for the R-matching procedure one requires explicit values for G_{21} and C_2. These can be determined from the calculation performed with EVENT2. First one determines G_{21} by fitting the $\alpha_s^2\ln(B)$ component of $I(B)$; then one subtracts it out and fits the asymptotically constant part of $I(B)$, which yields C_2. One obtains:

	B_T	B_W
G_{21}	1.988 ± 0.25	1.869 ± 0.25
C_2	2.330 ± 0.25	2.946 ± 0.25

It is necessary to go to extremely small $B \sim e^{-8}$ before a sufficiently asymptotic behaviour sets in. As a result the value of C_2 which one obtains is strongly dependent on the value taken for G_{21}. The results shown for C_2 were obtained by fixing G_{21} (at its central value), and the error is that purely from the fitting procedure for C_2. Roughly, the

[1]Note that the coefficients in [8] are given for $(\alpha_s/2\pi)^n$; this differs from the convention used here.

value of C_2 that would have been obtained with a different G_{21} would be shifted by $(-8 \pm 2)[G_{21} - G_{21}(\text{central})]$.

A final aspect of matching that needs to be mentioned is that one usually introduces a parameter B_{lim} at which the resummed calculation is constrained to vanish through a substitution of the form

$$\frac{1}{B} \to \frac{1}{B} - \frac{1}{B_{\text{lim}}} + 1\,. \tag{5.7}$$

For the purposes of fitting experimental results, a reasonable choice of B_{lim} is important; in the section that follows though, it well be set equal to 1 in order to facilitate the comparison with results that do not include matching.

5.3 The B_T distribution

To illustrate the changes introduced by our new calculation, we show in figure 2a the B_T distribution in the approximation that $\lambda = 2$ and in the case with the full treatment of the quark recoil. Examining first the case without matching, one sees that the distribution is shifted towards lower values of B, as expected. Associated with the shift is an increase in the height of the peak of the distribution at small B.

If one considers instead the results with log-R matching, one finds that the effect of going from $\lambda = 2$ to full recoil is much smaller.[2] The explanation is linked to the following property of log-R matching procedure: if one supplies it with the wrong forms for G_{22} and g_2, as long as those forms are consistent with each other, the input from the full 2-loop result reestablishes the correct value for G_{22}.

In figure 2a one notes that the matched and non-matched curves are quite significantly different. At large B this is to be expected, since one is beyond the range of validity of the resummed calculation. At small B the difference is less welcome. However there remain certain logarithmic exponentiated terms which have not been taken into account in our resummed calculation, in particular those that contribute to G_{21}. In figure 2b we plot our non-matched resummed calculation with the additional inclusion of the effect of G_{21} as determined numerically in the previous section. At small B this leads to very good agreement with the log-R matched results. At large B, as one expects, a significant difference remains. Figure 2b also shows a curve resulting from R-matching, and it is seen to be everywhere in good agreement with the log-R matching.

6 Conclusions

We have shown that a proper treatment of the contribution to B of the primary quarks recoiling against an ensemble of soft gluons is essential for predicting the B-distribution with single-logarithmic accuracy.

[2]In contrast it is not consistent to show results for $\lambda = 2$ with R-matching, because the R-matching procedure requires the input of values for G_{21} and C_2 — these cannot be reliably obtained if one uses a wrong value of G_{22}. The log-R matching procedure doesn't suffer from this problem because it doesn't require the explicit input of G_{21} and C_2.

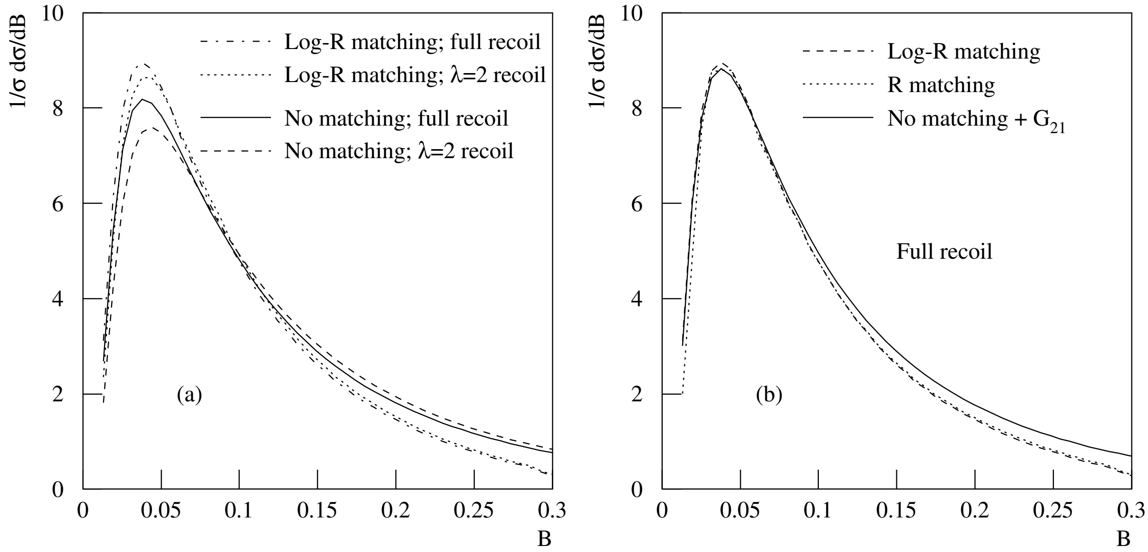

Figure 2: (a) The effect of going from the $\lambda = 2$ approximation for the quark recoil, to the full treatment of quark recoil, both in the case without any matching and in the case of log-R matching; (b) The effect of different matching procedures and a comparison with the case without any matching, but with the inclusion of the G_{21} term; for all curves, $\sqrt{s} = 91.2$ GeV and $\alpha_s(M_Z) = 0.118$.

We have verified that the evolution equation for the B-distribution derived by CTW embodies all the necessary ingredients to provide the resummed perturbative prediction with single-logarithmic accuracy. The improvement we made concerns the solution of this equation. We suggested two forms of the final result, (4.25) and (4.26). The former is the CTW-spectrum evaluated at a rescaled value of $B \to 2B/\lambda$, with λ a single-logarithmic function, $\lambda = \lambda(\alpha_s \ln 1/B)$ which decreases with B from 2 to 1. The latter form is the CTW-answer supplied with the single-logarithmic factor which does the same job of shifting the distribution to smaller B values. This tendency is opposite to that expected from the $1/Q$ power correction effect in the B-spectrum [3].

We were able to show the absence of the non-logarithmic contribution proportional to the coupling at the reduced momentum scale, $\alpha_s(BQ)$, which, if present, could damage the perturbative prediction at small values of B.

It should be noticed, however, that beyond the first order in $\alpha_s(BQ)$, given the present state of the art, such damage looks unavoidable. Indeed, consider the most interesting feature of the B-distribution which is its characteristic maximum at $B = B_{max} \ll 1$. A maximum emerges as a result of an interplay between the first-order peaked cross section, $\propto (\alpha_s \ln 1/B)/B$, and the all-order Sudakov suppression exponent, $\exp(-R(\lambda/2B)) \sim \exp(-\alpha_s \ln^2 1/B)$. The latter factor takes over, clearly, when $R'(1/B) \sim \alpha_s(BQ) \ln 1/B$ approaches unity. With B decreasing $\ln 1/B$ increases and so does the running coupling $\alpha_s(BQ)$. Formally speaking, in perturbation theory, that is for $Q \to \infty$, the expansion parameter $\alpha_s(BQ)$ stays small in the region of the maximum. However, in reality (and for any foreseeable energies) $\alpha_s(B_{max}Q)$ becomes numerically large. This undermines the reliability of the perturbative prediction for $B < B_{max}$ since the neglected subleading corrections of the order $\alpha_s^2 \ln 1/B$, and among those, $\alpha_s(BQ)R' \sim \alpha_s(BQ)$ are no longer numerically negligible. Corrections of this sort arise, in particular, from the hard-emission subtraction term $R' \sim \alpha_s \ln 1/B \to \alpha_s(\ln 1/B - 3/4)$ in the single-logarithmic pre-exponent.

To trigger genuine confinement effects, the $(A_1 \ln Q + A_2)/Q$ shift in the B-distribution [2, 3], it is tempting to look at smaller Q values. However, some care should be exercised here since the kinematical range of B shrinks. Moreover, one should bear in mind the above-mentioned intrinsic uncertainty of the perturbative prediction for $B < B_{max}$ which becomes larger for smaller values of Q. To be on a safe side, one should try to stay with the B-values to the right of the maximum, $B > B_{max}$.

It remains to be seen whether the $1/Q$ power correction extracted with the use of the improved perturbative expression derived in the present paper, will exhibit the expected $\ln Q$ enhancement.

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A One-gluon emission with single-logarithmic accuracy

The single gluon emission distribution is

$$\frac{d\sigma_1}{\sigma} = dx_1 dx_2 \frac{\alpha_s C_F}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \qquad x_1 = \frac{2pQ}{Q^2}\,, \quad x_2 = \frac{2\bar{p}Q}{Q^2}\,.$$

We introduce the gluon c.m. energy fraction z and transverse momentum k_t

$$z = 2 - x_1 + x_2\,, \qquad \frac{k_t^2}{Q^2} = \frac{(1-x_1)(1-x_1)}{1-z} \equiv \epsilon^2\,.$$

If the gluon is in the right hemisphere we have

$$x_1 = 1 - \tfrac{1}{2}\left(z + \sqrt{z^2 - 4(1-z)\epsilon^2}\right)\,, \quad x_2 = 1 - \tfrac{1}{2}\left(z - \sqrt{z^2 - 4(1-z)\epsilon^2}\right)\,.$$

If the gluon is in the left hemisphere these expressions are interchanged.

In terms of the gluon variables we have

$$\frac{d\sigma_1}{\sigma} = 2\,\frac{d^2k_t}{\pi k_t^2} dz\, \frac{\alpha_s C_F}{2\pi}\, \frac{1 + (1-z)^2 - 2(1-z)\epsilon^2}{\sqrt{z^2 - 4(1-z)\epsilon^2}}\,,$$

where the factor 2 takes into account that the gluon can be emitted in the right or left hemispheres. Integrating,

$$\int_{z_0}^{1} dz \frac{1 + (1-z)^2 - 2(1-z)\epsilon^2}{\sqrt{z^2 - 4(1-z)\epsilon^2}} = 2 \ln \frac{1}{\epsilon} - \frac{3}{2} - 4\epsilon^2 \ln \epsilon + \mathcal{O}\left(\epsilon^4\right)\,,$$

where z_0 is the zero of the square root and is given by $z_0 = 2\epsilon + \mathcal{O}(\epsilon^2)$. Then for small ϵ we can approximate the quark splitting function by replacing

$$\frac{1+(1-z)^2 - 2(1-z)\epsilon^2}{\sqrt{z^2 - 4(1-z)\epsilon^2}} \Rightarrow \frac{1+(1-z)^2}{z}\,\Theta(z - \frac{k_t}{Q})\,,$$

where the leading correction is of order $\epsilon^2 \ln \epsilon$. It produces a contribution $\mathcal{O}(\alpha_s(Q))$ to the integral of the gluon emission and is taken care of by the factor $C(\alpha_s(Q))$.

B The integral (4.15)

The coefficient $\Delta(\nu, b)$ of $R''(\mu)$ in the expansion of $\delta R(\nu, b)$ in (4.15) is given by

$$\begin{aligned}\Delta(\nu,b) &= -\frac{d}{d\epsilon}\left(\frac{1-(\mu/\nu)^{\epsilon}\,\Gamma(1+\epsilon)}{\epsilon}\right) + \ln\mu\ln\frac{\nu}{\mu} + c(\nu,b) \\ &= \tfrac{1}{2}\left(\ln\frac{\mu}{\nu} + \gamma_E\right)^2 + \frac{\pi^2}{12} + \ln\mu\ln\frac{\nu}{\mu} + c(\nu,b)\,,\end{aligned} \tag{B.1}$$

with $c(\nu, b)$ given by

$$c(\nu,b) = \int_0^\infty \frac{dx}{x}\ln x\, e^{-\nu x}\left(J_0(bx) - 1\right) = \ln\nu\ln\frac{\mu}{\nu} + \int_0^\infty \frac{dx}{x}\ln x\, e^{-x}\left(J_0(xb/\nu) - 1\right)\,. \tag{B.2}$$

This shows that $\Delta(\nu, b)$ depends only on the ratio μ/ν.

For the relevant integrals over b we use

$$\int_0^\infty \frac{\nu b db}{(\nu^2+b^2)^{3/2}} = \int_1^\infty \frac{dy}{y^2} = 1\,, \tag{B.3}$$

$$\int_0^\infty \frac{\nu b db}{(\nu^2+b^2)^{3/2}}\cdot\ln\frac{\mu}{\nu} = \int_1^\infty \frac{dy}{y^2}\ln\frac{1+y}{2} = \ln 2\,, \tag{B.4}$$

$$\int_0^\infty \frac{\nu b db}{(\nu^2+b^2)^{3/2}}\cdot\ln^2\frac{\mu}{\nu} = \int_1^\infty \frac{dy}{y^2}\ln^2\frac{1+y}{2} = \frac{\pi^2}{6} - \ln^2 2\,. \tag{B.5}$$

The inverse Fourier integral of $c(\nu, b)$ is

$$\begin{aligned}\int_0^\infty \frac{\nu b db}{(\nu^2+b^2)^{3/2}}\,c(\nu,b) &= \int b db \int p dp e^{-\nu p} J_0(bp)\int_0^\infty \frac{dx}{x}\ln x\, e^{-\nu x}\left(J_0(bx)-1\right) \\ &= \int_0^\infty \frac{dx}{x}\ln x\,\left[e^{-2\nu x} - e^{-\nu x}\right] = -\gamma_E\ln 2 + \tfrac{1}{2}\ln^2 2 + \ln 2\ln\nu\,.\end{aligned} \tag{B.6}$$

Assembling all terms, we obtain the result in the text.

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